Midterm, Econ 171, February 24, 2015

There are 6 questions. Answer as many as you can. Good luck!

**Problem 1.** Each of two sellers has one unit of an unusual commodity available to sell. Seller A posts a price of $40 for its item. Seller B posts a price of $60 for its item. There are exactly two potential buyers for this commodity. Each buyer would be willing to pay $100 to get one of these items and both buyers are aware that there are two buyers and each is willing to pay $100 for the item. Without knowing what the other buyer has done, the two buyers simultaneously decide which of the two sellers to approach. If two buyers approach the same seller, the seller tosses a fair coin to decide which of them to sell to. The other has to go home without making a purchase. We assume that a disappointed buyer is not able to approach the other seller. A buyer’s payoff if he obtains an item for price $p$ is $100-p$. Buyers seek to maximize their expected payoffs.

A) Show the strategic form of the game played between the two buyers.

B) Find all of the pure and mixed strategy Nash equilibrium profiles for this game. List the strategy of each player for any equilibrium that you find.
C) If the seller’s profit from the sale is the price of the item if at least one buyer approaches him and zero if no buyers approach him, what is the expected profit in Nash equilibrium of Seller A? What about Seller B?

D) Suppose that Seller B’s price is $50 and that Seller A’s price is \( p_A \). For what values of \( p_A \) is there a pure strategy Nash equilibrium for the game between the two buyers? For what values of \( p_A \) is the only Nash equilibrium for this game a mixed strategy equilibrium?

**Problem 2.** In the game, *Rock, Paper, Scissors*, two players simultaneously choose one of the strategies, Rock, Paper, or Scissors. If both choose the same strategy, both get payoff of zero. If one chooses Rock and the other chooses Scissors, the Rock player gets a payoff of 1 and the Scissors player gets -1. If one chooses Scissors and the other chooses Paper, the Scissors player gets a payoff of 1 and the Paper player gets a payoff of -1. If one chooses Paper and the other chooses Rock, the Paper player gets a payoff of 1 and the Rock player gets -1. Consider the following variant of this game. Player 1 is able to play any of the three strategies, but Player 2 can only play Rock or Paper.

A) Show this game in strategic form. Are there any strictly dominated strategies?
B) How many Nash equilibria does this game have? What is (are) the Nash equilibrium strategy profile(s)

C) Find the Nash equilibrium payoff for each player.

**Problem 3.** A) Define an independent, private values information setting for an auction.
B) Define a common values information setting for an auction.

C) In which of these settings, does the “winners curse” occur? Describe the mistake by bidders that would lead to the so-called winners curse?
Problem 4. Two players are involved in a dispute. Player 1 does not know whether Player 2 is strong or weak. Player 2 knows the strength of both players. Each player has two possible strategies, Fight or Yield. A player who chooses Yield will get a payoff of 0 regardless of the other player’s strength or action. If a player chooses Fight and the other player chooses Yield, then the payoffs will be 100 for the player who chose Fight and 0 for the other player. If both players choose Fight, then if Player 2 is strong, Player 1 gets a payoff of −100 and Player 2 gets a payoff of 100. If both players choose Fight and Player 2 is weak, then Player 1 gets a payoff of 100 and Player 2 gets a payoff of -100. Suppose that Player 1 believes that with probability $p$, Player 2 is strong and with probability $1 - p$, Player 2 is weak.

A) Draw an extensive form representation of this game.

B) Find Bayes-Nash equilibrium profiles for both players if $p > 1/2$. What is the expected payoff to Player 1 in Bayes-Nash equilibrium?
C) Find Bayes-Nash equilibrium profiles for both players if $p < 1/2$. What is the expected payoff to Player 1 in Bayes-Nash equilibrium?

D) Suppose that before play begins, Player 1 has an opportunity to hire a spy, who will accurately report whether Player 2 is weak or strong. The cost of hiring the spy is $c$, so that if Player 1 hires the spy, his payoff, contingent on choosing either Fight or Yield will be reduced by $c$. What is the most that Player 1 would be willing to pay to hire spy if $p > 1/2$. What is the most that Player 1 would pay to hire the spy if $p < 1/2$? (Your answers will be functions of $p$.)
Problem 5. In the game below, the top number is Player 1’s payoff, the middle number is Player 2’s payoff, and the bottom number is Player 3’s payoff.

A) How many strategies are possible for Player 1? How many strategies are possible for Player 2? How many strategies are possible for Player 3? Give an example of a possible strategy for Player 2.

B) How many subgames does this game have? How many proper subgames does this game have?
C) For each proper subgame of this game, find all of the subgame perfect Nash equilibrium profiles.

D) Find all of the subgame perfect Nash equilibrium strategy profiles for this game.
Problem 6. Consider a first-price sealed bid auction in which there are two bidders. The bidders’ valuations for the object being sold are independently drawn by Nature. Each bidder’s valuation must take one of the three values 3, 5, or 8. The probability is .3 that a bidder’s value is 3, the probability is .3 that the bidder’s value is 5, and the probability is .4 that the bidder’s value is 8.

A) Determine whether there is a symmetric Bayes-Nash equilibrium in which a bidder bids 2 when her valuation is 3, bids 3 when her valuation is 5 and bids 4 when her valuation is 8. Explain how you reached your answer.
B) Determine whether there is a symmetric Bayes-Nash equilibrium in which a bidder bids 2 when her valuation is 3, bids 3 when her valuation is 5 and bids 5 when her valuation is 8. Explain how you reached your answer.